How much time necessary to photo-generate Fermi surface from true electron vacuum?

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Presence of Fermi surface is the base of solid state science.

BCS, Kondo, C(S)DW, Plasmon theories are all assumed, it has been already well established.
How much time necessary to photo-generate Fermi surface from true electron vacuum?
Motivation

Rapid relaxation dynamics of optically excited electrons in metallic systems, has already been widely investigated. In most cases, however, only a few electrons are excited, while, the main part of electrons is still in the ground state, works as an infinite heart reservoir, resulting in quite rapid relaxation of newly given energy and momentum.

What occurs, if a macroscopic number of electrons are excited, at once, into a truly vacant conduction band, without electronic heat reservoir?
Coulombic inter-electron scatterings within the conduction band, being completely elastic, can give no net energy relaxation.
Phonon relaxation, two time regions

Avalanching initially, but soon it slows down infinitely, as approaches Fermi degeneracy, since only low energy phonons are available.

Two pulse excitations of GaAs, InP by visible laser
Time resolved photo-emission spectrum of conduction band electron

Phonon relaxation, and photo-generation of Fermi degenerate phase
Most simple, but ultimate photo-induced phase transition from true electron vacuum.

Experiment by Kanasaki and Tanimura
Lattice structure of GaAs, InP

Energy bands


Experimental determination of two time regions

Avalanching

Critical slowing down

by Kanasaki and Tanimura

0.4 ps (InP)
1.0 ps (GaAs)

Fermi degeneracy
Many-electron and acoustic phonon coupled system ($\equiv H$)

$$H = H_0 + H_i,$$

$$H_0 \equiv H_e + H_p, \quad H_i \equiv H_{ep} + H_{ee},$$

$$H_e \equiv \sum_{k,\sigma(=\alpha,\beta)} (e(k) - \mu) a^+_k, a_{k,\sigma}, \quad 0 \leq e(k) \leq B, \quad B = 5 \text{ eV},$$

$$n_M \equiv (2N)^{-1} \sum_{k,\sigma(=\alpha,\beta)} a^+_k a_{k,\sigma}, \quad n_M = 0.001 \sim 0.005, \quad \mu \equiv \text{Chemical potential}$$

Electron-phonon coupling and Hubbard type weak Coulomb repulsion

$$H_p \equiv \sum_q \omega_q b^+_q b_q, \quad \omega_q \equiv c_s |q|, \quad 0 \leq \omega_q \leq \omega_M (\equiv 24 \text{ meV}), c_s = 30 \text{ Å/(pico. sec.)},$$

$$H_{ep} \equiv S(2N)^{-1/2} \sum_{q,k,\sigma(=\alpha,\beta)} (b^+_q + b_{-q}) a^+_{k-q,\sigma} a_{k,\sigma}, \quad S \approx 1 \text{ eV}$$

$$H_{ee} \equiv U \sum_{\ell} (n_{\ell,\alpha} - n_M) (n_{\ell,\beta} - n_M), \quad U \approx 1 \text{ eV},$$
\[ n_{\ell,\sigma} \equiv a_{\ell,\sigma}^{\dagger} a_{\ell,\sigma}, \quad a_{\ell,\sigma} \equiv (2N)^{-1/2} \sum_{k} e^{i k \cdot \ell} a_{k,\sigma}, \]

The whole electronic system is always in a plan wave state only around the bottom of the conduction band minimum, with only a low carrier density, well described by one-boy \( H_e \), and effects of \( H_i \) is weak.

Density matrix at a time \( t \) \( \rho(t) \)

\[ \rho(t) \rightarrow \rho_e(t) \rho_p, \quad \rho_p \equiv e^{-H_p/k_B T_p}, \quad T_p = 0K \]

Phonon system is always heat reservoir.

\[ < n_{\ell,\sigma}(t) > \equiv \text{Tr}(n_{\ell,\sigma} \rho(t)) / \text{Tr}(\rho(t)), \quad < \ldots > \equiv \text{Tr}(\ldots \rho(t)) / \text{Tr}(\rho(t)), \]

\[ < n_{\ell,\sigma}(t) > \rightarrow n_M, \quad \text{independent of time} \quad t \]

The first order effect is always absent

\[ < H_i > = < H_{ee} > = < H_{ep} > = 0 \]
I. Statistical relaxation theory, 
Electron temperature ($\equiv T_e(t)$) is always well defined.

At each time $t$, electron temperature $T_e(t)$ ($0 \leq T_e(t) \lesssim 200$K) is always well established in electronic system, prescribed by one-body $H_e$, due to intra-system multiple scattering by $H_{ee}$, but gradually decreases, as it releases its energy to the phonon system through $H_{ep}$. We can forget about $H_{ee}$, except $T_e(t)$

$$H_i \rightarrow H_{ep},$$

Density matrix is

$$\rho(t) \rightarrow \rho_e(t)\rho_p, \quad \rho_e(t) \equiv e^{-\frac{H_e}{k_B T_e(t)}}$$

Total energy decrease of electrons, due to temperature decrease from $T_e$ to $(T_e - \Delta T_e)$, $\rightarrow$ Electronic heat capacity ($\equiv C(T_e)$)

$$C(T_e) = \frac{\partial <H_e>}{\partial T_e}, \quad <H_e> = \sum_{k,\sigma(=\alpha,\beta)} (e_k - \mu) <n_{k,\sigma}>, \quad n_{k,\sigma} \equiv a_{k,\sigma}^{+}a_{k,\sigma}$$

Fermi distribution:

$$<n_{k,\sigma} > = \frac{e^{-(e(k)-\mu)/k_BT_e}}{1+e^{-(e(k)-\mu)/k_BT_e}},$$
where, $\mu(T_e)$ should be determined at given $T_e$ from the self-consistent condition

$$n_M = (2N)^{-1} \sum_{k,\sigma(=\alpha,\beta)} <n_{k,\sigma}>$$

Thus we get

$$\Delta <H_e(T_e) > = C(T_e) \Delta T_e, \quad C(T_e) \propto T_e,$$

which is well known to be linear at low temperature? Luttinger, PR 119(1960)1153.

**Total energy increase of phonon system through second order of $H_{ep}$, within a time interval $\Delta t$ from $t$.**

Time evolution of $\rho(t + \Delta t)$ from $\rho(t)$

$$\rho(t + \Delta t) = e^{-i\Delta t H} \rho_e(t) \rho_p e^{i\Delta t H}$$

$$<H_p(t + \Delta t) >= \text{Tr} \left( H_p e^{-i\Delta t H} \rho_e(t) \rho_p e^{i\Delta t H} \right) / \text{Tr} \left( \rho_e(t) \rho_p \right),$$

$$e^{-i\Delta t H} = e^{-i\Delta t(H_0 + H_i)} = e^{-i\Delta t H_0} \exp_+ \left\{ -i \int_0^{\Delta t} d\tau \tilde{H}_i(\tau) \right\},$$
\[ e^{i\Delta t H_0} e^{-i\Delta t H} = \exp_+ \left\{ -i \int_0^{\Delta t} d\tau \, \tilde{H}_i(\tau) \right\} \]

Here, the interaction representation \( \tilde{O} \) of an operator \( O \) is

\[ \tilde{O}(\Delta t) \equiv e^{i\Delta t H_0} O e^{-i\Delta t H_0} \]

Its straightforward expansion is

\[ \exp_+ \left\{ -i \int_0^{\Delta t} d\tau \, \tilde{H}_i(\tau) \right\} = 1 + (-i) \int_0^{\Delta t} d\tau_1 \, \tilde{H}_i(\tau_1) + (-i)^2 \int_0^{\Delta t} d\tau_1 \int_0^{\tau_1} d\tau_2 \, \tilde{H}_i(\tau_1) \tilde{H}_i(\tau_2) \]

\[ + (-i)^3 \int_0^{\Delta t} d\tau_1 \int_0^{\tau_1} d\tau_2 \int_0^{\tau_2} d\tau_3 \, \tilde{H}_i(\tau_1) \tilde{H}_i(\tau_2) \tilde{H}_i(\tau_3) + \ldots \]

Its complex conjugate
\[ e^{i\Delta tH} = e^{i\Delta t(H_0 + H_i)} = \exp_- \left\{ i \int_0^{\Delta t} d\tau' \tilde{H}_i(\tau') \right\} e^{i\Delta tH_0}, \]

\[ \exp_- \text{ negative chronologically ordered exponential from right to left.} \]

\[ e^{i\Delta tH} e^{-i\Delta tH_0} = \exp_- \left\{ i \int_0^{\Delta t} d\tau' \tilde{H}_i(\tau') \right\} \]

Its straightforward expansion is

\[ \exp_- \left\{ i \int_0^{\Delta t} d\tau' \tilde{H}_i(\tau') \right\} = 1 + (i) \int_0^{\Delta t} d\tau_1' \tilde{H}_i(\tau_1') + (i)^2 \int_0^{\Delta t} d\tau_1' \int_0^{\tau_1'} d\tau_2' \tilde{H}_i(\tau_2') \tilde{H}_i(\tau_1') \]

\[ + (i)^3 \int_0^{\Delta t} d\tau_1' \int_0^{\tau_1'} d\tau_2' \int_0^{\tau_2'} d\tau_3' \tilde{H}_i(\tau_3') \tilde{H}_i(\tau_2') \tilde{H}_i(\tau_1') + \ldots \ldots \ldots \]

Thus

\[ \rho(t + \Delta t) = e^{-i\Delta tH_0} \exp_+ \left\{ -i \int_0^{\Delta t} d\tau \tilde{H}_i(\tau) \right\} \rho_e(t) \rho_p \exp_- \left\{ i \int_0^{\Delta t} d\tau' \tilde{H}_i(\tau') \right\} e^{i\Delta tH_0}, \]
Non zero second order term at phonon vacuum, $\rho_p$ at $T_p$ ($= 0$ K)

$$< H_p(t + \Delta t) > = \frac{\text{Tr}\left( H_p \int_0^{\Delta t} d\tau_1 \tilde{H}_{ep}(\tau_1) \rho_e(t) \rho_p \int_0^{\Delta t} d\tau'_1 \tilde{H}_{ep}(\tau'_1) \right)}{\text{Tr}(\rho_e(t)\rho_p)}$$

$$= \int_0^{\Delta t} d\tau'_1 \int_0^{\Delta t} d\tau_1 \frac{\text{Tr}(\tilde{H}_{ep}(\tau'_1)H_p\tilde{H}_{ep}(\tau_1)\rho_e(t)\rho_p)}{\text{Tr}(\rho_e(t)\rho_p)} = \Delta t \int_{-\Delta t}^{\Delta t} dy \frac{\text{Tr}(\tilde{H}_{ep}(y)H_p\tilde{H}_{ep}(0)\rho_e(t)\rho_p)}{\text{Tr}(\rho_e(t)\rho_p)}$$

$$= \ldots \ldots \ldots ,$$

$\Delta t \rightarrow \infty$, long time limit

So called golden rule
\[
\int_{-\Delta t}^{\Delta t} d\gamma e^{-i\bar{\epsilon}(k-q)\gamma t} (1-\langle n_{k-q,\sigma} \rangle)e^{i\bar{\epsilon}(k)\gamma t} \langle n_{k,\sigma} \rangle \omega_q e^{-i\omega_q \gamma} \\
= 2\pi\Delta t\omega_q (1-\langle n_{k-q,\sigma} \rangle) \langle n_{k,\sigma} \rangle \delta(\omega_q + \bar{\epsilon}(k-q) - \bar{\epsilon}(k)) \\
= \Delta t \Gamma(T_e),
\]

**e-h recombination by phonon emission**

\[
\Gamma(T_e) = 2\pi \frac{S^2}{2N} \sum_{q,k, \sigma(=\alpha,\beta)} \omega_q (1-\langle n_{k-q,\sigma} \rangle) \langle n_{k,\sigma} \rangle \delta\left(\omega_q + e(k-q) - e(k)\right)
\]

**Energy conservation**

\[
C(T_e) |\Delta T_e| = \Gamma(T_e) \Delta t, \\
\frac{\partial T_e}{\partial t} = -\frac{\Gamma(T_e)}{C(T_e)}
\]

Thus, we finally get the equation for electronic system cooling.
At low temperatures \( \Gamma(T_e) \propto T_e^4 \)

Electron-hole pair number around \( e_F \) \( T_e \)

Acoustic phonon mode density \( T_e^2 \)

Acoustic phonon energy \( T_e \)

Energy

\( \mu(T_e) \)

Chemical potential

Phonon mode density

\( k_B T_e \)

\( \omega_q \)

\( \omega_{q_{\text{max}}} \)
While, total energy decrease of electrons, due to temperature decrease from $T_e$ to $(T_e - \Delta T_e)$, at low temperatures

$$[< H_e(T_e) > - < H_e(T_e - \Delta T_e) >] \propto T_e^2 \rightarrow T_e|\Delta T_e|$$

$$\text{energy} - e_F \times (\text{number of electron, or hole around } e_F)$$

$$k_B T_e$$

$$k_B T_e / e_F$$
Finally,

$$T_e |\Delta T_e| \propto \Delta t \ T_e^4, \quad \frac{\partial T_e}{\partial t} \propto -T_e^3, \quad T_e \propto t^{-\frac{1}{2}}$$

Slowing down of relaxation speed than exponential decay.

Power law, half-life can not be defined.
Iterative theory for real time relaxation dynamic without electron temperature approximation
We now recover the full interactions for multiple scattering.

\[ H_i = (H_{ep} + H_{ee}), \]

\[ \rho(t) \rightarrow \rho_e(t)\rho_p, \quad \rho_p \equiv e^{-\frac{H_p}{k_B T_p}}, \quad T_p = 0K \]

\[ \rho_e(t) \text{ is now non-equilibrium state starting from the photo-excitation.} \]

\[ \tilde{\rho}(t + \Delta t) = \exp_+ \left\{ -i \int_0^{\Delta t} d\tau \, \tilde{H}_i(\tau) \right\} \rho_e(t)\rho_p \exp_- \left\{ i \int_0^{\Delta t} d\tau' \, \tilde{H}_i(\tau') \right\} \]
Second order time evolution $\Delta t$ from the transient state at $t$

$$\tilde{\rho}(t + \Delta t) = \rho_e(t) \rho_p$$

$$+ \int_0^{\Delta t} d\tau_1 \tilde{H}_{ep}(\tau_1) \rho_e(t) \rho_p \int_0^{\Delta t} d\tau'_1 \tilde{H}_{ep}(\tau'_1)$$

$$- \int_0^{\Delta t} d\tau_1 \int_0^{\tau_1} d\tau_2 \tilde{H}_{ep}(\tau_1) \tilde{H}_{ep}(\tau_2) \rho_e(t) \rho_p - \rho_e(t) \rho_p \int_0^{\Delta t} d\tau'_1 \int_0^{\tau'_1} d\tau'_2 \tilde{H}_{ep}(\tau'_2) \tilde{H}_{ep}(\tau'_1)$$

$$+ \int_0^{\Delta t} d\tau_1 \tilde{H}_{ee}(\tau_1) \rho_e(t) \rho_p \int_0^{\Delta t} d\tau'_1 \tilde{H}_{ee}(\tau'_1)$$

$$- \int_0^{\Delta t} d\tau_1 \int_0^{\tau_1} d\tau_2 \tilde{H}_{ee}(\tau_1) \tilde{H}_{ee}(\tau_2) \rho_e(t) \rho_p - \rho_e(t) \rho_p \int_0^{\Delta t} d\tau'_1 \int_0^{\tau'_1} d\tau'_2 \tilde{H}_{ee}(\tau'_2) \tilde{H}_{ee}(\tau'_1)$$

What we want to know finally is the time evolution of electron number $n_{k,\sigma}(t + \Delta t)$

$$\frac{\partial n_{k,\sigma}(t+\Delta t)}{\partial \Delta t}, \quad n_{k,\sigma}(t + \Delta t) \equiv \langle n_{k,\sigma} \rangle$$
Rate equation for $n_{k,\sigma}(t)$

Gain of $n_{k,\sigma}(t)$ proportional to $(1-<n_{k,\sigma}>)$, loss proportional to $<n_{k,\sigma}>$.

$$\frac{\partial n_{k,\sigma}(t)}{\partial t} = (1-<n_{k,\sigma}(t)>)(\Gamma_{ep,k}^+(t) + \Gamma_{ee,k}^+(t))-<n_{k,\sigma}(t)>(\Gamma_{ep,k}^-(t) + \Gamma_{ee,k}^-(t))$$

$$\Gamma_{ep,k}^+ \equiv \pi S^2 N^{-1} \sum_q <n_{k+q,\sigma}> \delta(e(k) + \omega_q - e(k+q))$$

$$\Gamma_{ee,k}^+ \equiv 2\pi U^2 N^{-2} \sum_{k'} (1-<n_{k',-\sigma}>\sum_{k''} <n_{k-k'',\sigma}> <n_{k'+k'',-\sigma}> \delta(e(k) + e(k') - e(k-k'') - e(k'+k''))$$

$$\Gamma_{ee,k}^- \equiv 2\pi U^2 N^{-2} \sum_{k'} <n_{k',-\sigma}> \sum_{k''} (1-<n_{k+k'',\sigma}>)(1-<n_{k'+k'',-\sigma}>) \delta(e(k+k'') + e(k'-k'') - e(k) - e(k'))$$

Iterative theory for dynamics

$$n_{k,\sigma}(t + \Delta t) = n_{k,\sigma}(t) + \Delta t\left(1-<n_{k,\sigma}(t)>\right)(\Gamma_{ep,k}^+(t) + \Gamma_{ee,k}^+(t)) - <n_{k,\sigma}(t)>(\Gamma_{ep,k}^-(t) + \Gamma_{ee,k}^-(t))\right)$$

$$n_{k,\sigma}(t + 2\Delta t) = n_{k,\sigma}(t + \Delta t) + \Delta t\left(1-<n_{k,\sigma}(t + \Delta t)>\right)(\Gamma_{ep,k}^+(t + \Delta t) + \Gamma_{ee,k}^+(t + \Delta t)) - <n_{k,\sigma}(t + \Delta t)>(\Gamma_{ep,k}(t + \Delta t) + \Gamma_{ee,k}(t + \Delta t))\right)$$
Residual photo-excitation energy ($\equiv \Delta E_r$) at final stage

$$\Delta E_r \propto (\Delta k_F)^2$$
Final stage of phonon relaxation

Transition rate \((\equiv \Gamma)\), \(\Gamma \propto (\Delta k_F)^4\)

Electron – hole number \(\propto \Delta k_F\)

Phonon mode density \(\propto (\Delta k_F)^2\)

Phonon energy \(\propto \Delta k_F\)
Finally

\[(\Delta k_F)^2 \propto \Delta t (\Delta k_F)^4, \quad \frac{\partial (\Delta k_F)}{\partial t} \propto (\Delta k_F)^3, \quad \Delta k_F \propto t^{-\frac{1}{2}}\]

Slowing down of relaxation speed than exponential decay.
Relaxation dynamics of residual excitation energy $\Delta E_r$, Theory
Two time regions

Avalaching

$\propto \frac{1}{\text{Time}}$

$\propto (\Delta k_F)^2$

Critical slowing down

Residual energy $\Delta E_r$ (eV)

Time after photo-excitation (ps)
Experimental determination of two time regions

Avalanching

by Kanasaki and Tanimura

0.4 ps (InP)
1.0 ps (GaAs)

Critical slowing down

Fermi degeneracy
Avalanching speed rapidly increases as electron density \((k_F)\) increases.

\[
\Gamma_{ep,k}^+ \propto k_F \omega
\]

Available phonon energy \(\omega\) also increases as \(k_F\) increases.
How much time necessary to photo-generate Fermi surface from true electron vacuum?

Never terminates.
Initial stage Coulombic elastic scattering

\[ \frac{\frac{1}{2} E^2 n(E)}{\int E^2 n(E) dE} \]

\[ \frac{n_M U^2}{B} \]

\[ n_M \lesssim 10^{-3}, \text{ rare than e-ph} \]
Decay channels

1. Radiative recombination of e-h pair, \(10^{-9}\) sec

2. Momentum, charge and spin fluctuations give no energy dissipation

3. Auger recombination of e-h pair with no energy dissipation, \(10^{-12}\) sec

4. Inter-band Coulomb scattering, similarly to the intra-band one, gives no dissipation.
Importance of thermodynamic boundary condition

Uncontrolled boundary condition gives uncontrolled experimental results.
Plasmon

is the coulombic anti-bound state between electron-hole, above the well-established Fermi distribution.