Applying Global Optimization in Structural Engineering

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“If the only tool you have is a hammer,
the whole world looks like a nail.” - Mark Twain

Our tool is optimization

Is structural engineering a nail?
"Dwarfing visitors, the 70-foot-tall Corliss steam engine powered the 1876 Centennial Exposition's entire Machinery Hall. Built by George H. Corliss, it was the largest steam engine in the world. Of engines like the Corliss, William Dean Howells wrote, 'In these things of iron and steel the national genius speaks.'"
- www.150.si.edu/chap4/4ngin.htm
Abstract

A typical modern office building is supported by steel columns and beams arranged in bays (horizontally) and stories (US) (vertically). The structure must support static (weight) and dynamic (storms and earthquakes) loads, at modest construction costs. If the structure fails under extreme conditions, we want to control its failure. For example, we prefer failures that can be repaired, and we prefer an inward collapse to toppling over. Members under extreme loads exhibit multiple modes of failure, which must be understood and modeled.

Increasingly, software tools used by practicing structural engineers augment or replace engineering experience and rules-of-thumb by careful mathematical modeling and analysis to support rapid exploration of the design space. Optimization and nonlinear systems problems abound, and their reliable solution is life-critical. Many problems have nonlinear finite element formulations. Parameter values are known approximately, at best. Problems such as beam buckling are extremely sensitive to initial conditions. Problems such as selection of suitable members are discrete because we want to specify members from a catalogue in stock. Some problems have broad, flat minimal regions, and some admit continua of solutions. Are we having fun yet?

This talk is accessible to anyone who remembers how to solve calculus max-min problems in two variables. I assume no structural engineering beyond the fact that the lecture hall has not collapsed. I report a little on work that has been done, but mostly speculate on opportunities. Is your hammer in your hand?
References


References


Objectives: Buildings & Bridges

Fundamental tenet of good engineering design:

**Balance performance and cost**

Minimize weight and construction costs

While

- Supporting gravity and lateral loading
- Without excessive connection rotations
- Preventing plastic hinge formation at service load levels
- Preventing excessive plastic hinge rotations at ultimate load levels
- Preventing excessive lateral sway at service load levels
- Preventing excessive vertical beam deflections at service load levels
- Ensuring sufficient rotational capacity to prevent formation of failure mechanisms
- Ensuring that frameworks are economical through telescoping column weights and dimensions as one rises through the framework

From Foley’s NSF proposal
Risk-Based Optimization

E.g.: Performance vs. earthquake?

Minimize initial cost of construction for the structural system

While

• Ensuring a tolerable level of risk against collapse from a 2,500 year ground motion
• Ensuring a tolerable level of risk of not being able to occupy the building immediately after a 100 year event

Need

• Assemble ground motion time histories
• Define damage states for structural components (beams & columns)
• Define damage states for nonstructural components (walls)

Computational Issues

Modeling
Large nonlinear systems
Constrained global optimizations
Uncertain parameters
Desire certainties:
• Guaranties of performance
• Legal liabilities

Do you want to plead:

“Yes, Your Honor, we were aware of more reliable modeling methods and tools, but we didn’t use them.”

1. One Structural Element: Buckling Beam

C-shaped beam - thin wall steel member

Under ultimate load, how does it fail?

Modes:
  • Torsional (twisting)
  • Flexural (bending)
  • Local
  • Distortion

Image: David H. Johnson, Channel Buckling Test and FEA Model, Penn State. http://engr.bd.psu.edu/davej/wwwdj2.html

1. One Structural Element: Buckling Beam

![Graph showing buckling behavior and stress vs. half-wavelength relationship.]

- **Local Distortional**
- **Euler (torsional)**
- **Euler (flexural)**

Dimensions:
- 34mm
- 64mm
- 8mm
- t=0.7mm
2. Building Structure: Failure Modes
2. Building Structure: Failure Modes

Buckling (failure) modes include
- Distortional modes (e.g., segments of the wall columns bulging in or outward)
- Torsional modes (e.g., several stories twisting as a rigid body about the vertical building axis above a weak story)
- Flexural modes (e.g., the building toppling over sideways).

Controlling mode of buckling flagged by solution to eigenvalue problem

\[(K + \square K_g) \mathbf{d} = 0\]

- \(K\) - Stiffness matrix
- \(K_g\) - Geometric stiffness - e.g., effect of axial load
- \(\mathbf{d}\) - displacement response

“Bifurcation points” in the loading response are key
3. Simple Steel Structure
3. Simple Steel Structure: Uncertainty

Linear analysis: \( K \mathbf{d} = \mathbf{F} \)

Stiffness \( K = f_K(E, I, R) \)
Force \( F = f_F(H, w) \)

- \( E \) - Material properties (low uncertainty)
- \( I, A \) - Cross-sectional properties (low uncertainty)
- \( w_{DL} \) - Self weight of the structure (low uncertainty)
- \( R \) - Stiffness of the beams’ connections (modest uncertainty)
- \( w_{LL} \) - Live loading (significant uncertainty)
- \( H \) - Lateral loading (wind or earthquake) (high uncertainty)

Approaches: Monte Carlo, probability distributions

Interval finite elements: Muhanna and Mullen (2001)
“Uncertainty in Mechanics Problems – Interval Based Approach”
Muhanna & Mullen: Element-by-Element

Reduce finite element interval over-estimation due to coupling
Each element has its own set of nodes
Set of elements is kept disassembled
Constraints force “same” nodes to have same values

3. Simple Steel Structure

![Diagram of a simple steel structure with labeled components: $W_{DL}$, $W_{LL}$, $R^L_{con}$, $E_b$, $I_b$, $R^L_{con}$, $E^R_{col}$, $I^R_{col}$, $A^L_{col}$, $A^R_{col}$, and $H$.](image)
3. Simple Steel Structure: Nonlinear

\[ K(d) \cdot d = F \]

Stiffness \( K(d) \) depends on response deformations
Properties \( E(d), I(d), \) & \( R(d) \) depend on response deformations
Possibly add geometric stiffness \( K_g \)

 Guarantee bounds to strength or response of the structure?

 Extend to inelastic deformations?

 Next: More complicated component: Truss
4. Truss

Example from Muhanna, Mullen, & Zhang, Penalty-Based Solution for the Interval Finite Element Methods, DTU Copenhagen, Aug. 2003.
Examples – **Stiffness Uncertainty**

- **Two-bay truss**
- **Three-bay truss**

\[ E = 200 \text{ GPa} \]
Examples – **Stiffness Uncertainty 1%**

➢ Three-bay truss

Three bay truss (16 elements) with 1% uncertainty in Modulus of Elasticity, \( E = [199, 201] \) GPa

<table>
<thead>
<tr>
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<th>( \text{V2(LB)} )</th>
<th>( \text{V2(UB)} )</th>
<th>( \text{U5(LB)} )</th>
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<tbody>
<tr>
<td>Comb ( \Downarrow \text{10}^{-4} )</td>
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<tr>
<td>Over estimate</td>
<td>-0.011%</td>
<td>0.021%</td>
<td>0.025%</td>
<td>-0.015%</td>
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</table>
Examples – Stiffness Uncertainty 5%

➢ Three-bay truss

Three bay truss (16 elements) with 5% uncertainty in Modulus of Elasticity, $E = [195, 205]$ GPa

<table>
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<th>V2(LB)</th>
<th>V2(UB)</th>
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<tbody>
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<td>Comb $\pm 10^{-4}$</td>
<td>-5.969223</td>
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<tr>
<td>Over estimate</td>
<td>-0.321%</td>
<td>0.596%</td>
<td>0.933%</td>
<td>-0.634%</td>
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</tbody>
</table>
Examples – **Stiffness Uncertainty 10%**

➢ **Three-bay truss**

Three bay truss (16 elements) with 10% uncertainty in Modulus of Elasticity, $E = [190, 210] \text{ GPa}$

<table>
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<th></th>
<th>V2(LB)</th>
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<td>Comb $\pm 10^{-4}$</td>
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<td>-1.623%</td>
<td>2.862%</td>
<td>4.634%</td>
<td>-3.049%</td>
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</table>
5. 3D Steel Structures
5. 3D: Uncertain, Nonlinear, Complex

Complex? $N_{bays}$ and $N_{stories}$

3D linear elastic analysis of structural square plan:

$$6 \times (N_{bays})^2 \times N_{stories} \text{ equations}$$

Solution complexity is $O(N_{bays}^6 \times N_{stories}^3)$

**Feasible** for current desktop workstations for all but largest buildings

But consider

- Nonlinear stiffness
- Inelastic analysis
- Uncertain properties
- Aging
- Dynamic - $\bullet(t)$
- Beams as fibers
- Uncertain loads
- Maintenance
- Imperfections
- Irregular structures
- Uncertain assemblies

Are we having fun yet?
6. Dynamic Loading

Performance vs. varying loads, windstorm, or earthquake?

Force $F(x, t)$?

Wind distributions?
	- Tacoma Narrows Bridge
	- Milwaukee stadium crane
	- Computational fluid dynamics

Ground motion time histories?
	- Drift-sensitive and acceleration-sensitive
	- Simulate ground motion

Resonances?
	- Marching armies break time
	- Not with earthquakes. Frequencies vary rapidly

Moré: Optimization Is Central?

That’s the analysis part: Given a design, find responses

Optimal design?
  e.g., 8,000 inelastic analyses vs. $10^{13}$ combinations

Minimize cost
  s.t. Safety

Current practice: “Confidence parameter,” genetic algorithms
  Much domain knowledge: e.g., group members,
  intelligent mutation, object-oriented
Lessons in Applied Mathematics?

Vocabulary
Respect the experts
Ask questions
Listen
Start small
Who will buy?

Monarch Corliss Engine near Smithville, TX
- From Vintagesaws.com
Challenges

Life-critical - Safety vs. economy
Multi-objective optimization
Highly uncertain parameters
Discrete design variables
  e.g., 71 standard column shapes
  149 AISC standard beam shapes
Extremely sensitive vs. extremely stable
Solutions: Multiple isolated, continua, broad & flat
Need for powerful tools for practitioners

“Well, I’ve got a hammer.” - Peter, Paul, & Mary