Status of Study of Spin Dynamics in Electrostatic Rings to Search Electric Dipole Moment

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Our goal

Track particle with initial horizontal spin polarization for a very large number of orbits, say $10^9$, to detect the appearance of a vertical spin component that will indicate the presence of an electric dipole moment. EDM of proton $d_p < 10^{-26}\text{e} \cdot \text{cm}$. So we need a storage ring which will conserve horizontal spin polarization for a long time.
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Spin coherence time

**SCT**

Spin coherence time (SCT) — time when RMS spin orientation of the bunch particles reaches one radian.

Requirements of planned SrEDM experiment: the SCT should be more than 1000 seconds. During this time each particle performs about $10^9$ turns in storage ring moving on different trajectories through the optics elements.
Electric dipole moment
Spin dynamics simulation
Methods to increase SCT

\[
\frac{d\vec{S}}{dt} = \mu \vec{S} \times (\vec{B} - c\beta \times \vec{E}) + d \vec{S} \times (\vec{E} + \beta c \times \vec{B})
\]

\(\mu, d\) — magnetic and electric dipole moments, \(c\) is the speed of light, \(\beta\) is the relative velocity and \(E, B\) — the electric and the magnetic field vectors.

\[
\frac{d\vec{S}}{dt} = \vec{\omega}_G \times \vec{S}
\]

\[
\vec{\omega}_G = -\frac{e}{m_0\gamma c} \left\{ Gc\vec{B} - \left( G - \frac{1}{\gamma^2 - 1} \right)(\vec{\beta} \times \vec{E}) \right\}
\]

\[
G = \frac{g - 2}{2},
\]

\(G\) — the anomalous magnetic moment, \(g\) is the gyromagnetic ratio and \(\omega_G\) is the spin precession frequency.
T-BMT equation

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Purely Electrostatic Ring

\[ \vec{\omega}_G = -\frac{e}{m_0 \gamma c} \left\{ \left( \frac{1}{\gamma^2 - 1} - G \right) \left( \vec{\beta} \times \vec{E} \right) \right\}. \]

Frozen Spin Method

Consider \( \gamma = \gamma_{mag} : \)

\[ \frac{1}{\gamma_{mag}^2 - 1} - G = 0, \]

and \( \omega_G = 0. \) Proton “magic” energy is 232 MeV.
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Spin oscillation of non-magic particle

If $\gamma \neq \gamma_{\text{mag}}$:

$$\vec{\omega}_G = -\frac{e}{m_0 \gamma c} \left\{ -2G \frac{\Delta p}{p} \left( \vec{\beta} \times \vec{E} \right) \right\}.$$  

Spin oscillation tune $\nu_{sz}$ satisfies:

$$S_z = S_{z0} \cos 2\pi \nu_{sz} n, \quad \nu_{sz} = \frac{e\bar{E}_x L_{cir}}{2\pi m_0 c^2 \gamma} G \frac{\Delta p}{p},$$

where $\bar{E}_x$ is the average value of the deflecting electric field. If $(\Delta p/p)_{\text{max}} = 10^{-4}$, then $\nu_{sz} = 1.588 \cdot 10^{-4}$, or $\text{SCT} = 6300$ turns $\approx 1\text{msec}$. 
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Using of RF cavity to increase SCT

With \((\Delta p/p) = (\Delta p/p)_{\text{max}} \cos (\nu_z \varphi)\) equation describing the oscillation of the spin:

\[
\frac{d^2 S_z}{d\varphi^2} + \left\{ \frac{e E_x L_{\text{cir}}}{2\pi m_0 c^2 \gamma} 2G \left( \frac{\Delta p}{p} \right)_{\text{max}} \cos (\nu_z \varphi) \right\}^2 S_z = 0.
\]

The spin vibrates within a very narrow angle \(\Phi_{\text{max}}\) with longitudinal tune \(\Phi \sim \Phi_{\text{max}} \sin (\nu_z \varphi)\). The value \(\Phi_{\text{max}} \sim (\nu_{sz}/\nu_z)^2\) depends on the frequency ratio.
Equilibrium energy level modulation

Figure: Phase trajectory in longitudinal plane for initial coordinate $x = 3\text{mm}, y = 0$.

If $(\Delta p/p)_{\text{max}} = 10^{-4}$ and the beam emittance $2\text{mm} \cdot \text{mrad}$, SCT will be $\sim 500$ sec.
The second order influence on spin

The spin tune in the second approach versus momentum:

$$\frac{d^2 S_z}{d\phi^2} + \left\{ \frac{eE_xL_{cir}}{2\pi m_0 c^2 \gamma} \left[ -2G \frac{\Delta p}{p} + \frac{1 + 3\gamma^2}{\gamma^2} G \left( \frac{\Delta p}{p} \right)^2 \right] \right\}^2 S_z = 0$$

and

$$\nu_{sz} = \frac{eE_x L_{cir}}{2\pi m_0 c^2 \gamma} \left\langle -2G \left( \frac{\Delta p}{p} \right) \cos(\nu_z \phi) + \frac{1 + 3\gamma^2}{\gamma^2} G \left( \frac{\Delta p}{p} \right)^2 \right\rangle m$$

$$\cdot \cos^2(\nu_z \phi) = \frac{eE_x L_{cir}}{2\pi m_0 c^2 \gamma} \frac{1 + 3\gamma^2}{\gamma^2} \frac{G}{2} \left( \frac{\Delta p}{p} \right)_m^2$$
Variation of spin tune

Assuming “magic” condition we define a variation of the spin tune through the finite differences up to second order:

$$\delta \nu_S = \frac{e}{2 \pi m_0 c^2} \delta \left( \frac{1}{\gamma^2 - 1} - G \right) L_{\text{orb}} E_x \frac{1}{\gamma} \left[ 1 + \frac{\delta L_{\text{orb}}}{L_{\text{orb}}} + \frac{\delta E_x}{E_x} + \gamma \delta \left( \frac{1}{\gamma} \right) \right],$$

$$\delta \left( \frac{1}{\gamma^2 - 1} - G \right) = -2G \frac{\Delta p}{p} + \frac{1 + 3\gamma^2}{\gamma^2} G \left( \frac{\Delta p}{p} \right)^2 + \ldots$$

$$\frac{\delta L_{\text{orb}}}{L_{\text{orb}}} = \alpha_1 \frac{\Delta p}{p} + \alpha_2 \left( \frac{\Delta p}{p} \right)^2 + \ldots$$

$$\frac{\delta E_x}{E_x} = -k_1 \frac{x}{R} + k_2 \left( \frac{x}{R} \right)^2 + \ldots$$

$$\gamma \delta \left( \frac{1}{\gamma} \right) = -\frac{\gamma^2 - 1}{\gamma^3} \left( \frac{\Delta p}{p} \right) + \frac{(\gamma^2 - 1)^2}{2\gamma^5} \left( \frac{\Delta p}{p} \right)^2 + \ldots$$
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Variation of spin tune

Grouping the coefficients we have:

\[
\delta \nu_S = \frac{e L_{\text{orb}} E_x}{2 \pi m_0 \gamma c^2} G \left\{ -2 \frac{\Delta p}{p} + \left( \frac{\Delta p}{p} \right)^2 \left[ \frac{5 \gamma^2 - 1}{\gamma^2} - 2 \alpha_1 - \frac{1 + 3 \gamma^2}{\gamma^2} k_1 \frac{x}{R} + \frac{1 + 3 \gamma^2}{\gamma^2} k_2 \left( \frac{x}{R} \right)^2 \right] + 2 \frac{\Delta p}{p} \left[ k_1 \frac{x}{R} - k_2 \left( \frac{x}{R} \right)^2 \right] \right\}
\]
Variation of spin tune

Finally we have

$$\delta \nu_S = \frac{eL_{\text{orb}}E_x}{2\pi m_0 \gamma c^2} G \left[ F_2 \left( \alpha_1, k_1, k_2, \frac{x}{R} \right) \left( \frac{\Delta p}{p} \right)^2 + 2F_1 \left( k_1, k_2, \frac{x}{R} \right) \frac{\Delta p}{p} \right]$$

Or the spin tune equation can be represented in another form:

$$\delta \nu_S = \frac{eL_{\text{orb}}E_x}{2\pi m_0 \gamma c^2} G \left[ \tilde{F}_2 \left( k_2, \frac{\Delta p}{p} \right) \left( \frac{x}{R} \right)^2 + 2\tilde{F}_1 \left( k_1, \frac{\Delta p}{p} \right) \frac{x}{R} + \tilde{F}_0 \left( \alpha_1, \frac{\Delta p}{p} \right) \right]$$
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Figure: Two dimensional parabolic dependence of spin tune aberration on $(\frac{\Delta p}{p})^2$ and $(\frac{x}{R})^2$
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Figure: Two dimensional parabolic dependence of spin tune aberration on 
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Cylindrical deflectors

We studied how the SCT depends on initial conditions in a structure without optimization with $k_1 = 1$, $k_2 = -1$. Figure shows the maximum spin deflection angle after $10^9$ turns for different initial horizontal deviations.

![Graph showing the maximum spin deflection angle for different initial horizontal deviations.](image)
Cylindrical deflectors

The maximum angle for spin particles with initial $x = 2\text{mm}$ can be up to 10 radians even at the momentum deviation $\Delta p/p = 10^{-4}$. It means that spin of the particle will make $\approx 1.5$ turnovers during this time, or RMS spin deviation is about 2 radians, which corresponds to a SCT about 500 seconds.
Methods to increase SCT

Our goal is to achieve maximum flatness in the working range of the beam parameters.

- The first method is to fit the parameters of electrical deflector and ring lattice in order to reduce this dependence that is to choose the lattice with a compensation of the mutual influence of all parameters. In other words, we need to make the surface maximally flat in the workspace of $(\Delta p/p)^2$ and $(x/R)^2$.

- The second method is to alternately change the deflector parameters and thereby to alternate spin rotation.
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$k_1 = 0.94, \ k_2 = 0.96$

**Figure:** Maximum spin deflection angle after $10^9$ turns versus $x$ deviation

Maximum spread spin angle for the entire beam is less than 1 radian, or RMS deviation is about 0.2 radians, which corresponds to SCT about 5000 seconds.
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\[ k_1 = 0.94, \quad k_2 = 0.96 \]

Problems for customly shaped deflectors:
- How to create plates with the required \( k_1 \) and \( k_2 \)
- How to adjust optics to the required minimum SCT
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Problems for customly shaped deflectors:

- How to create plates with the required $k_1$ and $k_2$
- How to adjust optics to the required minimum SCT
The ring is equipped with two types of deflector with $k_1 = \text{const}$, and $k_2 = k_{av} \pm \delta k$ changing from one deflector to another.

- In such optics is easier to achieve minimum spin aberration
- Raising the field strength between the plates in even deflectors and reducing in the odd deflectors it effectively adjust the required coefficients $k_1$ and $k_2$. It allows to adjust the spin of aberration to minimum.
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Thank you for your attention.