SOME PROBLEMS
OF NANOPROBE MODELING

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1. Description of the algorithm for nanoprobe modeling
2. Some undesirable effects in nanoprobe
3. Description of fringe fields
4. Several modeling functions for fringe fields simulation
5. Nanoprobe requirements
6. Load curves distortion induced by fringe field effects
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The Logic Scheme of a Modeling Process

- The piecewise constant model for steering fields and the corresponding solution in linear case.
  - The previous stage + holding tolerances.

- The piecewise constant model for steering fields and solution in nonlinear case.
  - The previous stage + holding tolerances;
  - Fringe fields effects and the corresponding solution in linear case.
  - Fringe fields effects and the solution in nonlinear case.
  - The previous stage + holding tolerances.
  - Fringe fields effects, solution in nonlinear case, holding tolerances and correction of parasitic nonlinear effects.
Some Undesirable Effects in Nanoprobe

1. Nonlinear effects
   A. Spherical aberrations
   B. Shift aberrations
   C. Chromatic aberrations
   D. Rotation aberrations

2. Fringe fields effects
Description of Fringe Fields

Fringe field of single magnet lens

Fringe field segmentation

Ordinary conditions

\[
\begin{align*}
    f_{\text{left}}(s_0) &= f_{\text{right}}(s_3) = 0, \\
    f_{\text{left}}(s_1) &= f_{\text{right}}(s_2) = f_0, \\
    \frac{f_{\text{left}}(s_0)}{ds} &= \frac{f_{\text{right}}(s_1)}{ds} = \frac{f_{\text{left}}(s_2)}{ds} = \frac{f_{\text{right}}(s_3)}{ds} = 0.
\end{align*}
\]

Piecewise approximation of fringe field

\[
f(s) = \begin{cases} 
    f_{\text{left}}(s), & s \in [s_0, s_1), \\
    1, & s \in [s_1, s_2), \\
    f_{\text{right}}(s), & s \in [s_2, s_3). 
\end{cases}
\]
Description of Fringe Fields

Fringe field superposition example

Effective length of magnet lens

\[ L_{\text{eff}} = \frac{1}{k_{\text{max}}} \int_{s_0}^{s_3} k(s) \, ds = \gamma L_{\text{iron}}. \]

Asymptotic conditions

\[
\begin{align*}
\lim_{s \to +s_0} f_{\text{left}}(s) &= 0, & \lim_{s \to -s_1} f_{\text{left}}(s) &= 1, \\
\lim_{s \to +s_0} \frac{df_{\text{left}}(s)}{ds} &= 0, & \lim_{s \to -s_1} \frac{df_{\text{left}}(s)}{ds} &= 0, \\
\lim_{s \to +s_2} f_{\text{right}}(s) &= 1, & \lim_{s \to -s_3} f_{\text{right}}(s) &= 0, \\
\lim_{s \to +s_2} \frac{df_{\text{right}}(s)}{ds} &= 0, & \lim_{s \to -s_3} \frac{df_{\text{right}}(s)}{ds} &= 0.
\end{align*}
\]
Some Modeling Functions for Fringe Fields

A strictly increasing function

\( F(s) \) is defined on \((s_1, s_2)\)

Reflection transformation

\[
F_r(s) = F(-s + s_1 + s_2).
\]

Sewing together

\[
F_1(s) = \frac{F(2s)}{2},
\]

\[
F_2(s) = -\frac{F(2(-s + s_2 - s_1))}{2} + F(s_2).
\]
Some Modeling Functions for Fringe Fields

A model function for a)

\[ F(s) = \alpha^2 \left( \frac{1}{2} - \cos^2 \alpha s + \frac{3}{4} \tan^2 \alpha s \right). \]

A model function for b)

\[ F(s) = -\left( \frac{n(n-1)}{s^2} + m^2 s^{2m-2} + m(m+2n-1)s^{m-2} \right). \]

A model function for a)

\[ F(s) = -\alpha^2 \left[ n - n(n-1) \tan^2 \alpha t \right]. \]

A model function for b)

\[ F(s) = \frac{n(2m+1-s^2)}{4s^2} - n(n-1) \left( t - 2m \frac{H_{m-1}(s)}{H_m(s)} \right)^2. \]
Custom Model Function for Fringe Fields

![Graph showing F1 and F2 functions]

- $F_1$
- $F_2$

The graph illustrates the behavior of $F_1$ and $F_2$ as a function of $S$. The $x$-axis represents $S$, ranging from 0 to 0.1, and the $y$-axis represents the magnitude of the functions, ranging from 0 to 1.
Nanoprobe Requirements

«Symmetry on power supply» condition for «russian quadruplet»
\[ k(s) = -k(s_t - s), \quad s \in [s_0, s_t], \]
where \( s \) - a length parameter, \( k(s) \) - a distribution function of gradient along the optical axis of the system.

All length parameters should be more than technical values and more than zero.

Minimization of linear demagnification is 0.01 or even less

«Round to round» particle beam condition

\[ R(s_t|s_0) = R_g M R_a, \]
where \( R_g \) and \( R_a \) are matrizants for drift spaces,
\[ M = \begin{pmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{pmatrix}, \]
\[ m_{11} = m_{44}, \quad m_{12} = m_{34}, \]
\[ m_{21} = m_{43}, \quad m_{22} = m_{33}. \]

«Point to point» condition leads to
\[ g = \frac{a m_{11} + m_{12}}{a m_{21} + m_{22}}. \]
Load curves distortion for fringe field parts are equal to 1/32 and 1/16. Dashed line is corresponding load curve when left and right parts of fringe field are equal zero. Blue line conforms to 1/32 part of $L_{\text{eff}}$ for left and right parts, red line - to 1/16.

Load curves distortion for fringe field parts are equal to 1/8, 1/4 and 1/2. Dashed line conforms to load curve with zero fringe field parts, green line - to 1/8 part of $L_{\text{eff}}$ for left and right fringe fields, blue line - to 1/4 and red line to 1/2.
Particle Beam Demagnification

With fringe field part increasing final particle beam characteristics are growing too. It was considered 3 optimal working point with demagnification in piecewise constant model:

\[ DM \approx 0.004 \quad \text{for black line} \]
\[ DM \approx 0.0057 \quad \text{for blue line} \]
\[ DM \approx 0.047 \quad \text{for red line} \]
in both planes \( \{x, s\} \) and \( \{y, s\} \).

Plots are constructed with
\[ f_{\text{left}} = f_{\text{right}} = \frac{1}{32} \text{L}_\text{eff} \]
Improvement of nanoprobe parameters

Initial system parameters are following (initial demagnification $DM \approx 0.0042$ in both planes):

$L_{eff} = 1$, $k_1 = 1.415$, $k_2 = 0.6075574$, $a = 135.5625$, $\lambda_1 = 3.3$, $\lambda_2 = 5.5$, $g = 0.4705207552$.

It is necessary to move working point to $k_1 = 1.415$, $k_2 = 0.627152$ in order to obtain demagnification $DM \approx 0.0083$ in both planes instead of $DM_x \approx 7.603139989$ and $DM_y \approx 1.490232745$. 
Recommendations and Conclusion

1. After obtaining a set of optimal solutions it is required to drop too sensitive points.

2. Fringe fields effects should be considered because they could heavily impact on beam characteristics.

3. It is required to move working point from initial load curve to other load curve which was constructed with taking into account fringe fields effects in order to improve nanoprobe characteristics.

4. Nonlinear effects and aberrations influence could be decreased using correction procedure with multipoles.

5. It is necessary to realize comparative analysis of many effects in nanoprobe and struggle against the most essential parasitic effects.
Thanks for your attention!