On beam dynamics optimization problems

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Optimization of program and perturbed motions

\[ \frac{dx}{dt} = f(t, x, u) \quad \text{(1)} \]
\[ \frac{dy}{dt} = F(t, x, y, u) \]

(2)
\[ x(0) = x_0 \]
\[ y(0) = y_0 \in M_0 \]
\[ t \in T_0 = [0, T] \subset \mathbb{R}^1 \]

\[ H = \{I_i(X(t), U(t), t)\} \]

, where
\[ I_i = \int_{0}^{T} \phi_i(t, x(t), u(t))dt + g_i(x(T)) \quad \text{or} \quad I_i = \int_{0}^{T} \phi (w_1(t))dt + G(w_2) \]

(3)
\[ w_1(t) = \int_{M_i,u} \phi_i(t, x(t), y_i, u(t), \rho (t, y_i)dt \quad w_2 = \int_{M_i,u} g_i(y_T, \rho (t, y_T)dy_T \]

\[ H_{con} = \{I_i[\leq \neq <= ]k_i(\epsilon_i) | I_i \in H, k_i \in \mathbb{R}^1, \epsilon_i \in \mathbb{R}^1\} \]

(4)
\[ I = \sum c_i \cdot I_i \rightarrow \min \]
Objective:
To search solutions satisfying the specified requirements

Problems:
- Intellectual optimization and search
- Automated management of the optimization process
Optimization methods

- Gradient method
- Method of conjugate gradients
- Newton's Method
- BFGS method (quasi-Newtonian)
- Simplex method
- Hook-Dzhivs’s method
- Heavy ball method
Analytical representation of antigradient

\[
\text{gradI}(u) = \left\{ \frac{\partial H_1(t, x, \chi, u)}{\partial u} + \int_{M_{t,u}} \frac{\partial H_2(t, x, y, \rho, w_1, \mu, v, u)}{\partial u} \, dy_t \right\}, \quad \text{where}
\]

\[
H_1(t, x, \chi, u) = \chi^* \cdot f(t, x, u) - \phi_1(t, x, u)
\]

\[
H_2(t, x, y, \rho, w_1, \mu, v, u) = \mu^* \cdot F(t, x, y, u) + v \cdot \text{div}_y F(t, x, y, u) - \Phi'(w_1) \cdot \varphi_2(t, x, y, \rho, u)
\]

\[
\frac{d\chi}{dt} = -\frac{\partial f^*}{\partial x} \chi + \frac{\partial \phi_1^*}{\partial x} - \int_{M_{t,u}} \left( \frac{\partial F^*}{\partial x} \mu + v \frac{\partial \text{div}_y F^*}{\partial x} \right) dy_t + \Phi'(w_1) \int_{M_{t,u}} \frac{\partial \phi_2^*}{\partial x} dy_t
\]

\[
\frac{d\mu}{dt} = -\left( \frac{\partial F}{\partial y} + E \cdot \text{div}_y F \right)^* \mu - v \frac{\partial \text{div}_y F^*}{\partial y} + \Phi'(w_1) \frac{\partial \phi_2^*}{\partial y}
\]

\[
\frac{dv}{dt} = -v \cdot \text{div}_y F + \Phi'(w_1) \cdot (\varphi_2 - \rho \frac{\partial \phi_2}{\partial \rho})
\]

\[
\chi(T) = -\frac{\partial g_1(x(T))}{\partial x}^*, \quad \mu(T) = -G'(w_2) \frac{\partial g_2(y_T, \rho_T)}{\partial y},
\]

\[
v(T) = -G'(w_2) \cdot (g_2(y_T, \rho_T) - \rho_T \frac{\partial g_2(y_T, \rho_T)}{\partial \rho})
\]

, where \( G'(w_2) = \frac{dG(w_2)}{dw_2} \), \( \Phi'(w_1) = \frac{d\Phi(w_1(t))}{dw_1} \)
Charged particles dynamics in equivalent traveling wave

\[
\frac{db}{d\tau} = \frac{4eU_L}{W_0L} \cdot \cos(Kz) \cdot \cos(\vec{\omega} \cdot \tau + \phi_s) = \frac{4eU_L}{W_0L} \cdot [\cos(\vec{\omega} \cdot \tau - Kz + \phi_s) + \cos(\vec{\omega} \cdot \tau + Kz + \phi_s)] \tag{9}
\]

\[
\begin{cases}
\frac{d}{d\tilde{\tau}}(L/L_0)^2 = 2k \cdot \eta(\tilde{\tau}) \cdot \cos \phi_s(\tilde{\tau}) \\
\psi'' + \frac{(L/L_0)^2}{(L/L_0)^2} \psi' + \frac{(L/L_0)'}{(L/L_0)''} \psi - \frac{\eta(\tilde{\tau})}{(L/L_0)^2} \cdot (\cos \phi_s(\tilde{\tau}) - \cos(\psi + \phi_s(\tilde{\tau}))) = 0
\end{cases} \tag{10}
\]

, where \( \eta = \frac{(U_L\theta)}{(U_L \theta)_{\text{max}}} \quad k = \frac{\Omega_0}{\vec{\omega}} \quad \tilde{\tau} = \Omega_0 \tau \quad \Omega_0^2 = \frac{4\pi e(U_L \theta)_{\text{max}}}{W_0L^2} \)

\[
x' = 2\xi
\]

\[
z' = -\left(\frac{2\xi \cdot z}{x} + \frac{\xi' - \xi^2}{x} \right) \frac{y}{x} - \frac{\eta}{x} \left[ \cos \phi_s - \cos(\psi - \phi_s) \right] \tag{11}
\]

, where \( \xi = k \cdot \eta(\tilde{\tau}) \cos(\phi_s(\tilde{\tau})) \quad \xi' = k \cdot (\eta(\tilde{\tau})' \cdot \cos(\phi_s(\tilde{\tau})) - \eta(\tilde{\tau}) \cdot \phi_s(\tilde{\tau})' \cdot \sin(\phi_s(\tilde{\tau}))) \)
Beam parameters

- Output energy
- Defocusing factor
- Mean-square value of phase
- Mean-square value of beam length
- Monotonicity of change of mean-square value of phase
- Monotonicity of change of mean-square value of beam length
- “Capture” condition

\[ x(T) = b_f^2 \]
\[ A_{\text{def}} = \frac{2k^2\eta |\sin \varphi_s|}{x} \leq 0.01 \]
\[ \int_{M_{t,u}} (\psi^2 \rho) d\psi_t d\psi_u \]
\[ \int_{M_{t,u}} (\psi^2 x \cdot \rho) d\psi_t d\psi_u \]
\[ \frac{d}{dt} \int_{M_{t,u}} (\psi^2 \rho) d\psi_t d\psi_u \leq 0 \]
\[ \frac{d}{dt} \int_{M_{t,u}} (\psi^2 x \cdot \rho) d\psi_t d\psi_u \leq 0 \]
\[ |\psi(t)| \leq \psi_{\text{max}}(t) \]
System structure

User Interface → Problem definition → The subsystem of logical deduction

User Interface

Common knowledge base (static knowledge)
   Knowledge Base for individual cases (dynamic knowledge)

The subsystem explain decisions

Algorithmic solutions

User Interface

Problem definition

DB

Developer
Optimization process scheme

- The choice of the initial control
- The choice of the optimization method
- The choice of weight coefficients of funcionals
- Restrictions processing
- Analysis of results
Generation of the initial control

- The selection of a set of conditions found in the knowledge base
- The genetic algorithm
  - Encoding functions values
  - Encoding functions derivatives
The genetic algorithm

1. An initial population creation
2. Calculation of fitness functions for populations individuals (estimation)
   • (The beginning of the cycle)
     1. Selection of individuals from the current population (breeding)
     2. Interbreeding and / or mutation
     3. Calculation of the fitness functions for all of the individuals.
     4. The formation of a new generation
     5. If the stopping conditions are fulfilled then (the end of the cycle), otherwise (the beginning of the cycle).

Encoding of the function \( f(x) \) in the genome:

- To build a grid of \( x \) (n cells).
- To create a vector \( Y \) equal, \( f(x_i) \) or \( f'(x_i) \) (depending on the method of coding)
- To encode each element \( Y \) by the following rules

\[
  \text{genom}_i = \left[ \frac{Y_{i} - Y_{\text{min}}^\phi}{Y_{\text{max}}^\phi - Y_{\text{min}}^\phi} \cdot \text{Size}_{\text{gen}} \right]
\]  

(13)
Selecting of the coefficients. Neural network.

Terms of training and testing of the algorithm correctness:

- functionals contributions should be one order
- taking into account the priorities of the functionals (the problem)
- If \( I_i < \varepsilon \) then \( c_i = 0 \)

The activation function:
- sigmoid
  \[
  OUT = \frac{1}{1 + e^{-NET}}
  \]  
- hyperbolic tangent
  \[
  OUT = \frac{e^{NET} - e^{-NET}}{e^{NET} + e^{-NET}}
  \]
Selecting the optimization method

1. Set the rate of method selecting of equal 1
2. Evaluate the rate of optimization methods from point X
3. Adjust the appropriate rate
4. Correct weights of the optimizing functional
5. Choose the method M with a maximum likelihood descent and implement descent
6. The rate of method selection <a (defined parameters)
7. Modify the condition task
8. Desired result
9. Check the condition accordance with the original objectives
10. Implement the necessary modifications
11. The problem's initial conditions are achieved
12. Finish
Methodology of orientation in the set of controls

- Control clustering on a variety of search options (meaning functionals parameters of the model)

- In the space of existing solutions find ones that are closest to the specified conditions (in a given neighborhood)
Work Genetic algorithm results:

Synchronous phase

Intensity accelerate

Phase of charge particle
System optimization result:

Synchronous phase

Intensity accelerate

Phase of charge particle
Results:

- Genetic algorithm with derivative encoding is the most effective to produce initial control.
- Developed expert system allows to solve assigned problem in the automatic mode.
Thank you