Effects of the wiggler on the Hefei Light Source (HLS) storage ring

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I. Introduction of the Hefei Light Source
Introduction of the Hefei Light Source (HLS)

- HLS is a second generation synchrotron radiation light source in the National Synchrotron Radiation Lab in the University of Science and Technology of China in Hefei, China.

Figure: The position of Hefei in China, from google map
HLS is a second generation synchrotron radiation light source in the National Synchrotron Radiation Lab in the University of Science and Technology of China in Hefei, China.

Figure: The storage ring hall (outside and inside)
Introduction of the Hefei Light Source (HLS)

- HLS is composed of a 200 MeV linac and an 800 MeV storage ring. One wiggler and one undulator were installed.
- The storage ring consists of four triple bend achromat (TBA) cells and four 3 m long straight sections.
- HLS currently runs in general purpose operation mode (GPLS) with an emittance of $133 \text{ nm} \cdot \text{rad}$.

Figure: $\beta$ functions and dispersion function in GPLS mode

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A high brilliance light source (HBLS) operation mode lattice design was proposed, in which the tunes were (5.2073, 2.5351) and the emittance was 27 nm \cdot \text{rad}.

The insertion devices were expected to be able to run in HBLS mode.

We’ll discuss the effects of the wiggler in the HBLS mode.

**Figure:** $\beta$ functions and dispersion function in HBLS mode
II. The magnetic field model of the wiggler
The magnetic field model of the wiggler

- Cosy infinity 9.0 beam physics package
- The midplane field in the wiggler is described as
  \[ B_m(x, z) = B_0 \cos \left( \frac{2\pi}{\lambda} \cdot z + k \cdot z^2 \right), \]
- Considering the fringe field, \( B = K_l \cdot B_m \cdot K_r \), where
  \[ K_{l,r} = \frac{1}{1 + \exp(a_1 + a_2(\mp z \pm z_{l,r})/d + \cdots + a_{10}((\mp z \pm z_{l,r})/d)^9)} \]
- The wiggler is represented by
  \[ WI < B_0 > < \lambda > < L > < d > < k > < I > < A > \]
The magnetic field model of the wiggler

- The wiggler in HLS has single period with three poles. The peak magnetic field strength of the central pole and the poles in either side are 6 T and 4.3 T respectively.
- In COSY, we set
  \[ \lambda = 210\text{mm}, \; k = 0, \; B_0 = 6.08\text{T}, \; d = 40\text{mm}, \; l = 252\text{mm}, \; a_0 = 0.478959, \; a_1 = 1.911289, \; a_2, a_3, \ldots, a_{10} = 0. \]

Figure: The magnetic field of the wiggler by measurement (left) and by fitting (right)
Generate the 3D magnetic field from the field in the reference axis in the midplane.

Maxwell’s equation

\[ \nabla \cdot \vec{D} = \rho, \quad \nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}. \]

In our case

\[ \nabla \cdot \vec{D} = 0, \quad \nabla \times \vec{E} = \vec{0}. \]

\( \vec{B} \) has scalar potential \( V_B \), which satisfy

\[ \Delta V_B = 0, \]

and \( \vec{B} = -\nabla V_B \).
The magnetic field model of the wiggler

- Taylor expansion of $V_B$ in transversal coordinates $x$ and $y$.

$$V_B = \sum_{k=0}^{\infty} \sum_{l=0}^{\infty} a_{k,l}(z) \cdot \frac{x^k y^l}{k!l!}$$

- Then

$$\frac{\partial^2 V_B}{\partial x^2} = \sum_{k=0}^{\infty} \sum_{l=0}^{\infty} k(k-1) \cdot \frac{x^{k-2} y^l}{k!l!} \cdot a_{k,l}(z)$$

$$\frac{\partial^2 V_B}{\partial y^2} = \sum_{k=0}^{\infty} \sum_{l=0}^{\infty} l(l-1) \cdot \frac{x^k y^{l-2}}{k!l!} \cdot a_{k,l}(z)$$

$$\frac{\partial^2 V_B}{\partial z^2} = \sum_{k=0}^{\infty} \sum_{l=0}^{\infty} a''_{k,l}(z) \cdot \frac{x^k y^l}{k!l!}$$
From the Laplace equation, we get the following recursive relation,

\[ a_{k+2,l} + a_{k,l+2} + a''_{k,l} = 0 \]

\[ a_{k,l+2} + a''_{k,l} + a_{k+2,l} = 0 \]

and \( a_{k,0}(z) \) and \( a_{k,1}(z) \) can be chosen freely, and all the other coefficients are determined by them.

Computing the gradient of the potential, we get

\[ B_y(x, y = 0, z) = - \sum_k a_{k,1}(z) \cdot \frac{x^k}{k!} \]

\[ B_x(x, y = 0, z) = - \sum_k a_{k,0}(z) \cdot \frac{x^{k-1}}{(k-1)!} = 0 \]
The magnetic field components in the horizontal direction should be 0 in the $y = 0$ plane. So that $a_{k,0} = 0$ for all $k$, and all the other coefficients can be derived from $a_{k,1}$.

$a_{k,1}$ can be determined by the field in the reference axis, then the 3D field can be generated.

In our case, we assume the trace of the electrons are close to the reference axis and the width of the magnet is large enough, so that $B_y$ doesn’t depend on $x$, then $a_{0,1} = K_l * B_m * K_r$, and all the other $a_{k,1}$s are zero.
- Get the 3D magnetic field by DA method.
- Fix point problem

$$
\Delta V = \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2} = 0,
$$

$$
V = V|_{y=0} + \int_y \left\{ \frac{\partial}{\partial y} V|_{y=0} - \int_y \left( \frac{\partial}{\partial x} \left( \frac{\partial V}{\partial x} \right) + \frac{\partial}{\partial z} \left( \frac{\partial V}{\partial z} \right) \right) \right\}
$$

- In $y = 0$ plane, $\partial V / \partial y$ is just the field, and $V = y \cdot \partial V / \partial y$ since the field doesn’t explicitly depend on $y$.
- Iterate at most $n + 1$ times, we get the potential up to $n^{th}$ order. Then take the derivative to get the field.
III. The beam dynamic study by the COSY Infinity 9.0
In principle, COSY does the following things for tracking:

- Generate the map of each element to arbitrary order.
- Compose all the maps into an one-turn map.
- Get generating function from the one-turn map.
- Tracking by generating function.
The beam dynamic study by the COSY Infinity 9.0

Arbitrary order map from an ODE based on antiderivation operator

\[
\dot{\vec{z}} = \vec{f}(\vec{z}, t)
\]
\[
\vec{z} = \vec{z}_i + \int_{t_i}^{t} \vec{f}(\vec{z}, t')dt'
\]

- Fix point problem.
- Iterate \( n + 1 \) times, we get the expansion of \( \vec{z} \) in initial position \( \vec{z}_i \) and \( t \) up to \( n^{th} \) order.
Arbitrary order map from an ODE based on derivation operator

- For a given function in phase space \( g(\vec{z}, t) \), we have

\[
\frac{d}{dt} g(\vec{z}, t) = \vec{f} \cdot \nabla g + \frac{\partial}{\partial t} g = L_{\vec{f}} g, 
\]

where \( \dot{\vec{z}} = \vec{f} \).

- The Taylor expansion of the solution of the ODE is

\[
\vec{z}_f = \sum_{i=0}^{\infty} \frac{t^i}{i!} \cdot L_{\vec{f}}^i \cdot \vec{I}
\]

where \( \vec{I} \) is the identity function.
The beam dynamic study by the COSY Infinity 9.0

- When $g$ and $\vec{f}$ are not explicitly time dependant, the partial derivative respect to $t$ in $L_{\vec{f}}$ disappear. If $\vec{f}(\vec{0}) = \vec{0}$, let $g$ be a component of $\vec{z}$, we obtain the integrator.

$$\vec{z}_0 = \vec{z}_i, \quad \vec{z}_j = \frac{t \cdot (\vec{f} \cdot \vec{\nabla})}{i} \cdot \vec{z}_{j-1}, \quad \vec{z}_f = \sum_{j=0}^{\infty} z_j$$

- As to the elements, such as dipole, quadrupole etc, whose fields don’t depend on the time variable, the derivation method works well. As to the elements like wiggler whose field changes with the time variable, we can use the antiderivation method.
Set up the generating function from the map

- Take the 2nd kind of generating function as an example.

\[(\vec{Q}, \vec{p}) = (\vec{\nabla}_p F_2, \vec{\nabla}_q F_2)\]

- \(M = (M_1, M_2),\) \(M_1\) is the position part, \(M_2\) is the momenta part. Identity map \(I = (I_1, I_2)\).
- Let \(N_1 = (I_1, M_2),\) then \((\vec{q}, \vec{P}) = N_1(\vec{q}, \vec{p}), (\vec{q}, \vec{p}) = N_1^{-1}(\vec{q}, \vec{P})\)
- Let \(N_2 = (M_1, I_2),\) then \((\vec{Q}, \vec{p}) = N_2(\vec{q}, \vec{p}) = N_2 \circ N_1^{-1}(\vec{q}, \vec{P}) = G(\vec{q}, \vec{P})\)
- \(F_2\) is just the integration of \(G\).
Inverse map is a fix point problem.

- A given map \( A = A_1 + A_2 \), where \( A_1 \) is the linear part.
- \[
A \circ A^{-1} = (A_1 + A_2) \circ A^{-1} = I
\]
- \[
A^{-1} = A_1^{-1} \circ (I - A_2 \circ A^{-1})
\]

Symplectic tracking by generating function

- \((\vec{Q}, \vec{p}) = G(\vec{q}, \vec{P})\)
- Solve for \(\vec{P}\) from \(\vec{p} = G_2(\vec{q}, \vec{P})\).
- Get \(\vec{Q}\) directly from \(\vec{Q} = G_1(\vec{q}, \vec{P})\)
IV. The beam dynamic effects of the wiggler
The linear effects of the wiggler

The linear map is

\[
M_x = \begin{pmatrix}
0.9997516 & 0.2532518 \\
0 & 1.000248
\end{pmatrix},
\]

\[
M_y = \begin{pmatrix}
0.9409307 & 0.2462244 \\
-0.4852251 & 0.9358029
\end{pmatrix}.
\]

The tunes change from (5.2073, 2.5351) to (5.2072, 2.5674).

Figure: Tunes with and without the wiggler
The linear effects of the wiggler

- The vertical $\beta$ function is distorted.
- The changes of the tunes and the $\beta$ function can be compensated by adjusting the quadrupoles.

Figure: $\beta$ functions and dispersion function with the wiggler
The nonlinear effects of the wiggler

- The dynamic aperture is up to 26 mm in horizontal direction, and up to 9 mm in vertical direction. (The 10th order tracking of 5000 turns by COSY infinity 9.0 beam physics package.)

Figure: The dynamic apertures without the wiggler
The nonlinear effects of the wiggler

Figure: The dynamic aperture with the wiggler
The nonlinear effects of the wiggler

Figure: The dynamic aperture with the wiggler without the fringe field
Conclusions

- The COSY Infinity 9.0 beam physics package is a strong and convenient tool in beam dynamics study.
- Strong linear effects. The tunes and the vertical $\beta$ function need compensation.
- The dynamic apertures are only slightly affected.
- The wiggler could operate in the HBLS mode after compensation.